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## Modeling the Effects of Distance and Spatial Dependence in International Trade

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To the Graduate Council:

I am submitting herewith a thesis written by Jesse Oakes Piburn entitled "Modeling the Effects of Distance and Spatial Dependence in International Trade." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Geography.

Ronald V. Kalafsky, Major Professor

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Nicholas Nagle, Shih-Lung Shaw

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Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

**Modeling the Effects of Distance and Spatial Dependence in  
International Trade**

A Thesis Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Jesse Oakes Piburn

May 2013

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## **Dedication**

To my wife, Ashley

## **Acknowledgements**

I would like to thank my wife, Ashley, for supporting me throughout the writing of my thesis and my entire time in graduate school. Without her support I wouldn't have graduated and our apartment would be filthy.

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Finally, I'd like to thank Charlie for keeping my feet warm while I sat at the kitchen table writing this thesis. Woof.

## **Abstract**

The gravity model has been widely used estimate the effect that distance has in international trade; however, two important areas have seen little attention in the literature, namely, the influence of using a more accurate measure of distance and how distance effect estimates change when controlling for spatial dependence in observed trade flows. Using transportation networks to measure distance and estimating both a spatial lag and spatial error gravity model, Canadian provincial exports to the lower 48 states in the United States were analyzed to address these previously ignored issues. It was found that the traditional distance measure of great circle distance underestimated the distance between any given province and state pair by an average of 20%, however, it did so consistently across all pairs, therefore having no significant influence on the distance effect estimate, regardless of estimation technique. When trade was disaggregated into mode of transportation distance effect estimates differed significantly, reflecting the most efficient uses of each transport mode and also the commodities that flowed across them. Finally, when using the spatial lag gravity model, distance effect estimates decreased substantially compared to traditional least squares estimation, while the spatial error model provided asymptotically equivalent parameter estimates to least squares, but with overall increased predictive power.

## Table of Contents

Chapter 1 Introduction .....	1
1.1 The Role of Distance .....	1
1.2 Deficiencies in the Literature.....	2
1.3 Motivation of Study .....	3
Chapter 2 Literature Review.....	5
2.1 Gravity Model Background .....	5
2.2 Distance Effect in U.S.-Canadian Trade.....	8
2.3 Gravity Model and Spatial Effects.....	11
2.4 Spatial Econometric Gravity Model .....	14
Chapter 3 Data and Methods.....	17
3.1 Data.....	17
3.1.1 Trade Data.....	17
3.1.2 Spatial Weights Data.....	18
3.1.3 Modal Network Data.....	19
3.2 Methodological Background.....	23
3.2.1 Spatial Weights Matrix .....	24
3.2.2 Spatial Lag Model.....	25
3.2.3 Spatial Error Model.....	26
3.2.4 Notation and Ordering of Flows .....	28
3.3 Procedural Methodology.....	29
3.3.1 Non-Spatial Gravity Model.....	29
3.3.2 Generation of Gravity Model Spatial Weights Matrices .....	30
3.3.3 Spatial Lag Gravity Model.....	33
3.3.4 Spatial Error Gravity Model .....	36
Chapter 4 Results .....	40
4.1 Distance Measurement Results .....	40
4.2 Non-Spatial Gravity Model Results.....	41
4.3 Spatial Lag Gravity Model Results.....	43
4.4 Spatial Error Gravity Model Results.....	46
Chapter 5 Discussion and Conclusion .....	49
5.1 Distance Effect Estimates across Differing Measures of Distance.....	49
5.2 Distance Effect Estimates across Modes of Transportation .....	50
5.3 Distance Effect Estimates and Spatial Dependence.....	53
5.4 Conclusion .....	56
List of References .....	59
Appendix.....	67
Vita.....	71



## List of Tables

Table 1. Specifications of the Non-Spatial Gravity Model.....	30
Table 2. Specifications of the Spatial Lag Gravity Model.....	34
Table 3. Specifications of the Spatial Error Gravity Model .....	37
Table 4. Network Distance Measurement Increase .....	41
Table 5. Non-Spatial Gravity Model Results.....	42
Table 6. Moran's Index – Dependent Variable.....	43
Table 7. Spatial Lag Gravity Model Results.....	44
Table 8. Spatial Error Gravity Model Results .....	47
Table 9. Non-Spatial Gravity Model Results – 2008.....	68
Table 10. Spatial Lag Gravity Model Results – 2008.....	69
Table 11. Spatial Error Gravity Model Results – 2008 .....	70

## List of Figures

Figure 1. Cities Used for Distance Measurement .....	21
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## **Chapter 1**

### **Introduction**

The gravity model has been widely used to estimate the effect that distance has in international trade for the greater half of a century. At its most basic formulation, the gravity model states that the value of trade between an origin-destination (OD) pair is proportional to their economic masses and inversely proportional to the distance between them. Various specifications of the gravity model have been developed from this simple equation and fit the data surprising well; with an  $R^2$  averaging 0.7 across published literature (Baldwin & Taglioni, 2006) leading Chaney (2011) to call the gravity model one of the most stable and robust empirical models in economics.

#### **1.1 The Role of Distance**

Implicit in all specifications of the gravity model is the importance of spatial relationships in international trade – specifically, the role of distance. Distance has historically been one of the largest facilitators, and impediments, to international trade and because of this, it is important to obtain an accurate measure of just how much an effect distance has. While several authors have used the gravity model to attempt just that, the large base of empirical research using the gravity model in international trade has seen very little work that attempts to accurately measure the distance between OD pairs and have all but ignored Anselin and Griffith (1988) in their clarification on the ways in which standard econometric models fail to remain applicable for spatial data.

## 1.2 Deficiencies in the Literature

Although a main purpose of the gravity model is to estimate the effect distance has on trade, oddly, how this distance is measured has been virtually an afterthought in almost all of the previous literature. The standard approach to measuring distance between an OD pair has been to use great circle distance (M. A. Anderson & Smith, 1999; Baldwin, 1994; Deardorff, 1998; Helpman, Melitz, & Rubinstein, 2008; McCallum, 1995; Silva & Tenreyro, 2006; Wall, 2000). However, it is obvious that the routes goods are shipped across will not always follow this shortest path. Road and rail networks, for example, almost never follow a direct straight path and the resulting network distance can deviate dramatically from great circle distance. The effect of this is to underestimate the distance that separates trading partners in the gravity model literature and depending on the structure of this discrepancy could significantly change distance effect estimates.

In its standard least squares estimation the gravity model assumes independence among flows, an assumption that seems inappropriate for many spatial interactions. Known as the first law of geography, Tobler (1970, p. 236) suggests “everything is related to everything else, but near things are more related than distant things.” This deceptively simple statement is operationalized through spatial autocorrelation and if least squares estimation is used in its presence the resulting parameter estimates can suffer from bias and inefficiency (Anselin, 1988). While the assumption of independent flows has seen almost no debate in the international trade literature, it has long been questioned in the regional science community, where the gravity model has been widely

applied to the study of migration and transportation flows. Griffith (2007) provides an historical review of the regional science literature on this issue and credits Curry (1972) for being the first to hypothesize the presence of spatial dependence in spatial interaction flows; while later work, such as that by Griffith and Jones (1980), reiterates and further refines the idea that distance effect estimates are confounded by unacknowledged spatial autocorrelation.

Since international trade flows are inherently spatial interactions there is no reason for the *a priori* assumption that trade is an aberration of Tobler's first law and that each observation is independent of neighboring observations. However, this is the assumption that underlies the estimation of the gravity model in international trade and as previously mentioned, this theoretical misalignment also manifests itself empirically, generating concern not only from the theoretical perspective, but operationally, as well.

### **1.3 Motivations of Study**

These two shortcomings in the literature, namely, inaccurate distance measures and the lack of consideration for spatial dependence provide the motivation for this study. This study explores the empirical performance of the gravity model and the resulting distance effect estimate when using a more accurate measure of distance and controlling for spatial dependence through the use of modal network distance measures and spatial econometric methodology adapted from the regional science literature.

Specifically, this study asks:

1. Does the distance effect change when using a more realistic measure of distance?

2. Does the distance effect vary across mode of transportation?
3. Does the distance effect change when using estimation techniques that account for spatial dependence?

To answer these questions, spatial lag and spatial error gravity models were specified and applied to Canadian provincial exports to each of the contiguous states in the United States disaggregated into mode of transportation by adapting the spatial econometric modeling technique for OD flows originally developed by LeSage and Pace (2008) while using road and rail network datasets to calculate a more accurate measure of the distance between each OD pair. These road and rail network distances were then coupled with estimates for air transport distance to derive network weighted distance estimates for use in specifying spatial lag and spatial error gravity models for total exports.

The remainder of this paper is structured as follows. Chapter 2 consists of a review of relevant literature, including background on the gravity model in international trade, an overview of previous estimates of the distance effect, and the gravity model and spatial effects. Chapter 3 contains a description of the data including the network datasets, then an in-depth description of the methods used in the analysis, including brief overviews of both the spatial lag and spatial error models and then how they are extended to model spatial dependence in OD flows through Maximum-Likelihood (ML) estimation. Chapter 4 presents the results of the analyses; while lastly, Chapter 5 discusses the findings and recommends directions for future research.

## Chapter 2

### Literature Review

#### 2.1 Gravity Model Background

The inspiration for the gravity model comes from Newtonian physics and the law of universal gravity (Zhang & Kristensen, 1995), in which the attraction of two masses is directly proportional to the product of their masses and inversely proportional to the distance between them. Functionally:

$$\text{Force of Gravity} = G \frac{M_1 M_2}{(\text{dist}_{12})^2}$$

In trade, force of gravity is replaced with the value of trade, the gravitational constant  $G$  is analogous to the intercept constant,  $M_1$  and  $M_2$  are each trading partners' specific economic masses represented by individual economic characteristics, such as gross domestic product (GDP), population, GDP per capita, as well as other measures.

The standard approach to applying the gravity model in international trade is by using its log-normal functional form in an ordinary least squares estimation resulting in a model in matrix notation as shown below

$$\mathbf{y} = \alpha \mathbf{t}_n + \mathbf{X}_o \beta_o + \mathbf{X}_d \beta_d + \gamma \mathbf{d} + \boldsymbol{\varepsilon}$$

#### *Equation 1*

where  $\mathbf{y}$  is an  $n$  by 1 vector of logged trade flows,  $\mathbf{t}_n$  is an  $n$  by 1 vector of ones with  $\alpha$  denoting its constant parameter term,  $\mathbf{X}_o$  and  $\mathbf{X}_d$  are  $n$  by  $k$  matrices of  $k$  explanatory variables for each the origin and destination, respectively, logged unless otherwise specified, with  $\beta_o$  and  $\beta_d$  being their associated  $k$  by 1 parameter vectors,  $\mathbf{d}$  is an  $n$  by 1 vector of the logged distance between each OD pair with  $\gamma$  being its associated scalar

parameter. The disturbances are represented by the  $n$  by 1 vector  $\boldsymbol{\varepsilon}$  which is assumed  $\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I})$ .

The gravity model was first applied to international trade by Tinbergen (1962) and Poyhonen (1963). Since then, numerous authors have used varying specifications of the gravity model to estimate the trade effects of a wide variety of factors such as changes in the distance effect of time (Berthelon & Freund, 2008), trading blocs (Frankel, Stein, & Wei, 1998), currency unions (J. E. Anderson & Marcouiller, 1999), international borders (McCallum, 1995), intranational borders (Wolf, 2000), and common languages (Nitsch, 2000).

A common criticism of the gravity model was its lack of theoretical foundations arising from the fact the gravity model was used empirically before it was grounded theoretically in any prevailing trade theory (Leamer, 1994). Because of its simplicity and the inherent importance of geography in trade, however, the gravity model has subsequently been derived from several different theoretical foundations. The first accepted derivation was J. E. Anderson (1979) in which he derives the gravity model from a Cobb-Douglas style expenditure system with constant elasticity of substitution and countries with similar cost structures. Subsequent derivations include Bergstrand (1985) finding a gravity type model to arise from specialization as a result of an Armington structure of demand, Helpman (1987) constructing a gravity model from economies of scale, Davis (1995) using the Heckscher-Ohlin model and technological differences across countries to justify the gravity model, and Eaton and Kortum (2002) who develop a gravity equation from a modern adaptation of trade driven by Ricardian-



style comparative advantage. The Linder hypothesis (1961), that exports tend to reflect the home market thus trade arises between countries with similar market structures and per capital income, is a predominant theoretical derivation of the gravity model, as well (Bergstrand, 1990). Concentrating on the importance of location in trade, Asilis and Rivera-Batiz (1994) develop a theory of interregional trade in which spatial aspects play a central role. They formalize the role of distance by making location a dependent variable and examining how trade occurs through the interaction between areas, distance, and differing regional production configurations. Trade in this model occurs due to the dependent spatial separation of factors of production and labor. While the many theoretical beginnings of the gravity model may not always agree on the source of its origination, the one thing all derivations of the gravity model have in agreement, is that distance has an effect in determining the level of trade between any origin and destination. Just how much of an effect is a question of empirical investigation that numerous authors have attempted to answer.

The standard log-normal functional form of the gravity model allows distance coefficients to be interpreted as the elasticity of trade with respect to distance, for example, if the parameter coefficient for distance in a gravity model were estimated to be -0.72, this can be interpreted as given a 10% increase in the distance between trading partners, a 7.2% decrease in trade would be expected to occur. This conveniently allows for a viable comparison of the distance effect found throughout various studies. Leamer and Levinsohn (1995) compare previous distance effect estimates and claim an average distance effect of approximately -0.6. Overman, Redding, and Venables (2003) state that

the distance effect is normally estimated to be between -0.9 to -1.5, however their estimate only cites three previous studies to support this claim; those of Feenstra, Markusen, and Rose (2001), Frankel (1997), and Soloaga and Winters (2001). In a much more extensive and robust meta-analysis of 1,467 distance effects estimated throughout 103 papers, Disdier and Head (2008) find a mean distance effect of -0.91 and a median of -0.87, with 90% of estimates lying between -0.28 and -1.55. By this analysis, on average, a 10% increase in distance lowers bilateral trade by 9.1%. Of note is the fact that more than half of their samples are below the suggested interval by Overman et al. (2003).

## **2.2 Distance Effect in U.S.-Canadian Trade**

Of particular interest to this study are previous distance effect estimates obtained from gravity model analyses of Canadian provincial and U.S. state trade. While estimates of the distance effect obtained from other study areas provide a general indication of its influence, in regards to this study, previous distance effect estimates derived specifically from U.S.-Canadian trade provide a more applicable expectation. Providing this background is the research of McCallum (1995), M. A. Anderson and Smith (1999), and Wall (2000).

In one of the first U.S.-Canadian gravity model analyses, McCallum (1995) uses least squares to estimate a log-normal gravity model to study export trade between 30 U.S. states (the 20 largest by population, plus all states touching the U.S.-Canadian border) and the 10 Canadian provinces for 1988. In addition to distance and the GDPs of both origins and destinations, he includes a dummy variable set equal to 1 for all observances of interprovincial trade and 0 for all observances of trade that cross the U.S.-

Canadian border. This variable is interpreted as the border effect, or *ceteris paribus* how much more a given province trades with another province than a U.S. state. McCallum finds the distance effect to be -1.42, suggesting a 10% increase in distance results in a 14.2% decrease in trade. However, two aspects of this study should be addressed specifically as they are potential sources of bias in the estimated distance effect.

First, seven observations of zero trade flows between U.S. states and Canadian provinces are simply excluded because of the impossibility of including the natural log of zero in the regression analysis. These observations of no trade are between provinces and states that are relatively far from one another and simply excluding this substantive information has the effect of introducing bias into the estimates of the effect distance has on trade (Helpman, Melitz, & Rubinstein, 2008). The second potential source of bias in McCallum's distance effect estimate is that distance itself is measured with the standard approach of using great circle distance between largest cities, with two small adjustments made to this distance specification; for the cases of California and Texas distance is measured from the midpoint of the two largest cities. As mentioned previously, using great circle distance to measure the separation of an origin and destination often times underestimates the distance goods actually have to travel, resulting in inaccurate distance measures and potentially inaccurate estimates of the distance effect.

Similar to McCallum (1995), M. A. Anderson and Smith (1999) apply a least squares estimated gravity model to investigate U.S. state and Canadian provincial trade flows. What makes their investigation unique is that they not only estimate the border and distance effects for trade at an aggregate level, they estimate these effects on the

shipment of transportation equipment, in an effort to see how the international border and distance effect trade in an industry that has been largely free of tariff barriers since the 1965 U.S.-Canada Auto Pact. They find the distance effect for all shipments to be -1.28, while the distance effect on transportation equipment was -1.41. This increase in the distance effect is most likely the result of the highly regionalized U.S.-Canadian auto trade, which in 1990, the year in question, was still almost solely centered around Detroit and before many auto manufacturers migrated to the southeastern United States. Distance in this study is measured identically to the specification of McCallum (1995), thus introducing the same serial underestimation of the distance between origins and destinations.

Most recently in these specific studies of U.S.-Canadian gravity models, Wall (2000) again uses least squares to estimate a gravity model of U.S. state and Canadian provincial trade flows, this time for the years 1994 through 1996. Resolving the problem encountered by McCallum (1995), he includes observations of zero trade flows in his analysis by changing the standard dependent variable of the natural log of trade to the natural log of trade plus 1, allowing observations of no trade to be included in the regression as equal to 0 ( $\ln(1) = 0$ ). Wall uses four different specifications to model the border effect resulting in differing border coefficients. However, the distance effect stays consistent across all models, ranging between -1.12 to -1.15. Keeping with the standard approach of previous studies, distance is again measured using great circle distance between largest cities in each province and state with California and Texas being the only

exception, with distance being measured from the midpoint of the two largest cities in each state.

In the three gravity model analyses of U.S.-Canadian trade discussed above the estimated distance effect ranged from -1.12 to -1.42, implying that trade between U.S. states and Canadian provinces decreases by 11% to 14% when distance between the states and provinces increases by 10%. However, all of these estimates originate from a method of distance measurement that consistently underestimates the actual distance shipments have to travel and an estimation technique that fails to remain applicable in the presence of spatial dependence. Nevertheless, using great circle distance and least squares estimation has become the standard approach to gravity model analysis in international trade and very little work has been done to call into question its validity. Yet what little work that has been done by the international trade community, when coupled with the large base of spatial econometric literature gives reason to call into question these previous distance effect estimates and investigate further how they might change when spatial effects that are inherent to international trade flows are explicitly accounted for in the estimation procedure.

### **2.3 Gravity Model and Spatial Effects**

Despite the recognition of the importance of spatial effects when applying the gravity model in other fields such as migration, ecology, or agricultural economics, there has been very little research applying the gravity model to international trade that acknowledges, much less properly accounts for, the presence of spatial dependence in either the observed trade flows or the resulting error terms (Porojan, 2001). While these

two problems have not been completely ignored in the literature they have been given very little due diligence.

Baldwin (1994) discusses generic issues with the empirical implementation of least squares estimated gravity models such as data aggregation, zero trade flows, imports vs. exports, even distance measurement, but makes no mention of spatial effects.

Applying the gravity model to migration flows that exhibit heteroskedasticity, Flowerdew (1982) suggests the use of an iterative weighting method when using least squares estimation, which McCallum (1995) applies in his analysis, but makes no mention of the potential for spatially autocorrelated errors. In differentiating the effects of spatial proximity and regional trading blocs, Poon (1997) addresses the issue of “locational heterogeneity” by proposing a model with variable coefficients by applying Casetti’s (1972) expansion method to a gravity based model of trade. Acknowledging the possible presence of spatially autocorrelated error terms Bougheas, Demetriades, and Morgenroth (1999) use a Seemingly Unrelated Regression (SUR) to allow for correlation between error terms in their estimate of infrastructure impediments to trade in the European Union. However, for identical independent variables, OLS and SUR are identical and there is no gain in efficiency by using the alternative estimate (Greene, 2007). Burger, van Oort, and Linders (2009) point to three issues when using the log-normal least squares estimation of the gravity model; logarithm transformation bias, heteroskedasticity, and zero trade flows, and suggest an extension of the Poisson model specification put forward by Silva and Tenreyro (2006), but do not mention any possible

problems arising from spatial dependence in observed trade flows or the resulting error terms.

When the gravity model literature has attempted to consider the spatial relationships inherent to international trade, it is through the use simple dummy variables that do nothing to take into account spatial dependence in the estimation procedure. The adjacency variable used by Frankel et al. (1998) is an example of this practice. In their gravity model analysis Frankel et al. (1998) use a dummy variable for adjacency, equal to 1 if an OD pair share a common land border and equal to 0 if otherwise. In matrix form this dummy variable becomes identical in structure to that of a contiguity based, unstandardized specification of what is known as a spatial weights matrix; the workhorse of the spatial econometrics literature used to capture neighborhood influences on the variable of interest. However, in the form used by Frankel et al. (1998), the adjacency variable represents the influence of sharing a common border, without regard to how neighboring observations influence the value of trade.

In order to properly account for spatial dependence in regression analysis, one must look to models originating in the spatial econometrics literature, such as the widely used spatial lag and spatial error models described by Anselin (1988). These models use spatial weights matrices and special estimation techniques to properly model spatial dependence. These spatial weights matrices, however, are constructed to model dependence among  $n$  regions, resulting in a square  $n$  by  $n$  matrix. As a consequence, using these models in an origin-destination setting such as international trade, where all  $n$  regions act as origins each having as many as  $n - 1$  destinations, remained a stumbling

block; essentially preventing the gravity model literature from taking advantage of advantageous methods developed by the spatial econometrics literature. That is until LeSage and Pace (2008) bridged this gap and developed methodology to extend the traditional spatial weights matrix and corresponding spatial lag and spatial error models to properly model spatial effects in origin-destination flows and it is their methodological framework that was used as the foundation of the spatial lag and spatial error gravity models used in this study.

## **2.4 Spatial Econometric Gravity Model**

LeSage and Pace (2008) develop spatial weights structures which allow for the extension of the traditional spatial lag and spatial error models to account for spatial dependence in gravity model analysis and use this structure to specify a combination of three spatial weights matrices one each for origin, destination, and what they call origin-to-destination dependence. Their methodology has since been applied to patent citations (Fischer & Griffith, 2008), migration (LeSage & Fischer, 2010), and interregional commodity flows (LeSage & Llano, 2012), but to this author's knowledge has yet to be applied to international trade.

Intuitively, factors and economic forces such as natural resources, knowledge spillover, or agglomeration economies, that lead to trade flows from any origin to a particular destination may create similar flows from neighbors of that origin to the same destination, and the origin spatial weights matrix of LeSage and Pace (2008) captures this origin-based spatial dependence, whether it be observed as in the spatial lag gravity model or unobserved in the case of the spatial error gravity model. This operationalizes



the notion of Griffith and Jones (1980) that flows from an origin to a particular destination are enhanced or diminished in accordance to the flows from its neighboring origin locations to that destination.

Similar reasoning extends to the destination spatial weights matrix that is used to capture destination-based spatial dependence. For the same reasons why exports from neighboring origins may be similar, certain factors and economic forces that cause a destination to import from a particular origin may cause the neighbors of that destination to import similar flows from that origin. This phenomena is captured in the destination spatial weights matrix and puts into formulation the conclusion of Griffith and Jones (1980) that flows to a destination from a particular origin are enhanced or diminished in accordance to the attractiveness of its neighboring destinations.

The third spatial weights matrix used to capture, what LeSage and Pace (2008) call, origin-to-destination spatial dependence arises from the product of the origin and destination spatial weights matrices and reflects the average of flows from neighbors of the origin to neighbors of the destination.

The use of three separate spatial weights matrices allows for each of their influences to be quantified separately, a very important feature for modeling origin-destination flows. However, this gain in information comes at the loss of currently available software that is able to estimate a model with more than one spatial weights matrix. Consequently, if an author wishes to implement these methods they must develop their own algorithms to do so. Possibly contributing to the fact that LeSage and Pace's (2008) methodology has seen relatively little application, with the exception of the

authors' own subsequent work. For this study, model implementation and estimation were done using the R statistical computing language (R Core Team, 2012). A more detailed description of the implementation and estimation procedure follows in the next chapter.

## **Chapter 3**

### **Data and Methods**

This chapter defines the data and methods that are used for the analyses in this study. Section 3.1 provides a description on the sources of the data and how they were used, including the trade data, data for the generation of the spatial weights matrices, and modal networks. Following, Section 3.2 provides relevant background information on the spatial weights matrix, spatial lag model, and spatial error model. Once these methods and models are introduced, Section 3.3 describes how they are extended to develop the spatial lag and spatial error gravity models used in this study.

#### **3.1 Data**

##### **3.1.1 Trade Data**

The trade data that were used in this research were obtained from Statistics Canada, International Trade Division (table - CRO0130607). These included the value of exports by mode of transportation from each province of origin to each state of destination. The modes of transport were divided into rail, road, air, water, and other. All values were in current Canadian dollars.

Provincial gross domestic product data were obtained from Statistics Canada, CANSIM, table 384-0038 in current millions of Canadian dollars, while state gross domestic product data were obtained from the United States Bureau of Economic Analysis. BEA data were published in current millions of U.S. dollars. Because of this, they were converted to current millions of Canadian dollars using the official conversion rate published by the Bank of Canada, Canada's Central Bank. The conversion rate was

the monthly average rate for the month of October 2012, the month in which the trade data were extracted from Statistics Canada.

### 3.1.2 Spatial Weights Data

The spatial weights matrices used in the final analysis of this study follow the structure of those developed by LeSage and Pace (2008), which are discussed in detail in Section 3.2.6. However, traditional spatial weights matrices had to be generated before they could be transformed into those suitable for gravity model analysis. To generate these traditional spatial weights matrices, two shapefiles were downloaded from ESRI ArcGIS Online; one that contained all 50 U.S. states and the other being Canadian provinces and territories. Post processed in ArcMap 10.1 to insure data integrity, these shapefiles were edited to remove Alaska, Hawaii and the Canadian territories that were not included in this study. Then the dissolve tool was used based upon state/provincial name so to consolidate states/provinces that were represented as multiple features into a single feature in the attribute table and thus resulting in a single observation in the spatial weights matrix. At that point the integrate tool was used to insure that the states/provinces that have common boundaries were in fact spatially coincident in the data. Finally, the processed shapefiles were loaded in GeoDa (Anselin, Syabri, & Kho, 2006) where two, .gal file type, spatial weights matrices were created one representing each, the U.S. and Canada. Remaining in the traditional  $n$  by  $n$  form, these two spatial weights matrices would be appropriately transformed for gravity model analysis once they were imported into R. Details of this transformation are found in in section 3.3.2.

### 3.1.3 Modal Network Data

One of the main goals of this research is to see if the distance effect would change when using a more accurate measure of distance. The overwhelming majority of the previous literature, as detailed by Disdier and Head (2008) and implemented by studies such as; McCallum (1995), M. A. Anderson and Smith (1999), and Wall (2000) use the standard approach of great circle distance for measuring distance between all OD pairs. While this approach is appropriate for most trade transported via air, it underestimates, sometimes vastly, the actual distance that goods must travel when transported across rail, road, or water networks. Accordingly, in the attempt to answer if the distance effect would change when using a more accurate measure of distance, transportation network datasets were employed to better estimate the distance goods must travel when being shipped.

As mentioned in section 3.1.1, the trade data used in this study disaggregated total provincial exports into transport modes of rail, road, air, water, and other. Therefore from the outset this limited the networks that could be independently analyzed to rail, road, air, and water, since other is a catchall category with no further details provided in the dataset and of which its individual transport components cannot reliably be inferred. As detailed below, the transport modes of rail, road, and air were chosen to be studied independently and the resulting network distances were used to estimate a single, weighted total, distance for each OD pair based upon the value of trade transported across each mode in an OD pair's individual trading relationship. Referred to collectively as a network weighted distance, this distance measure reflects the effective distance that separates each

OD pair and provides a more accurate estimation of distance for use in analyzing total trade.

Before the network data is detailed below, Figure 1 provides the point locations for each province and state where distance was measured to and from. These are the same locations used by McCallum (1995), M. A. Anderson and Smith (1999), and Wall (2000) in their studies of U.S.-Canadian trade and were chosen for comparison purposes. The point locations are of the most populous cities in each province or state with two exceptions, in which California and Texas are measured from the mid-point of their two most populous cities.



*Figure 1. Cities Used for Distance Measurement*

The both the rail and road networks that were used in this research were from the North American Transportation Atlas Data (NORTAD) published by the Bureau of Transportation Statistics (BTS). Once these data were downloaded they were coupled with ESRI's network analyst extension and made into routable networks. From there the shortest path network distance between every origin and destination was calculated for both the rail and road networks.

For the rail network there were two cases which had to be handled differently. Prince Edward Island (PEI) and Newfoundland and Labrador (NL) do not have rail

networks that connect to the rest of Canada's railways. This is not a case where the data do not accurately reflect reality; PEI and NL simply do not have a rail connection to the other provinces. In these two cases the rail network distance that was included in the regression analysis was estimated as the great circle distance from the origin to each destination multiplied by the average percent increase in distance when comparing rail network distance to great circle distance for the other eight provinces. Essentially this takes a baseline distance (great circle) and increases it by the actual average percent increase in distance that was observed for the eight provinces that rail network distance could be established, an average increase of 21%. In effect, this allows for a pseudo network distance to be included in the analysis as a number other than zero which if not corrected could have significantly biased the distance effect estimates, because as is obvious the distance between PEI and Texas is not zero, regardless of mode of transportation.

Distance between origins and destinations for air transports were estimated as great circle distance. Great circle distance was thought appropriate as an estimate of air transportation distance for this research because none of the paths from origin to destination cross traditionally avoided airspace and none of the distances were far enough that it exceeded the maximum range of standard airliners. However, it should be noted that great circle distance may not always be a close approximation for international air transport and very much depends on the study area in question.

Trade that was classified as being transported by water, was not examined independently such as was done for trade by rail, road, and air, however these values



were included in the analysis of total trade. This was done in light of zero trade flows via water for most of the OD pairs, with obvious exceptions for coastal origins and destinations, such as Nova Scotia to New Jersey. Coupled with the lack of a comprehensive navigable waterway network dataset, it was decided to only include trade via water in the analysis of total trade.

For regressing total trade, a single measure of distance was needed that reflected an approximate distance that goods shipped from each origin to each destination had to travel regardless of the mode of transport. Using an individual network distance for total trade wouldn't be an accurate estimation since each OD pair differed in the make-up of its trade and by which mode it was transported. For example, using road network distance wouldn't be an accurate estimation if trade via road only consisted of 10% of total trade for a certain OD pair. Therefore, a network weighted distance was calculated for the use in analyzing total trade. For each OD pair its value of trade via rail, road, and air were each multiplied by their respective network distance, summed and then divided by the sum of its value of trade via rail, road, and air, resulting in a distance measure that is weighted based upon the network distances which are most prevalent in each trading relationship.

### **3.2 Methodological Background**

Since the procedural methodology used in this study reflects an extension of how the spatial weights matrix, spatial lag model, and spatial error model are classically structured in spatial econometrics, this section is meant to provide the appropriate background for readers not already familiar with their typical specifications. This is in

hopes that providing this background will allow a reader to better understand the ways in which gravity model analysis fits within the framework of spatial econometrics.

### 3.2.1 Spatial Weights Matrix

Formally, a spatial weights matrix,  $\mathbf{W}$ , is an  $n \times n$  positive matrix which specifies neighborhoods of theoretical influence for each observation. For each row  $i$ , a non-zero element  $w_{ij}$  defines  $j$  as being a neighbor of  $i$  and by convention, an observation is not a neighbor of itself (Anselin, 2002). For ease of comparison and interpretation, the spatial weights matrix is row standardized, so that the sum of each row is equal to 1. From this practice, each element in the spatial weights matrix is between 0 and 1, which allows the spatial lag operation to be taken as an averaging of the neighboring values (Cressie, 1993).

There are varying ways to define a neighbor in a spatial weight matrix, economic closeness, political closeness, physical distance, contiguity, as well as more general methods (Cliff & Ord, 1981). The most common method however is contiguity, with contiguity itself being broken into three main types; rook (only common boundaries), bishop (only common vertices), and queen (common boundaries and vertices) (Upton & Fingleton, 1985).

Further discussion on the specification of spatial weights matrices used for this study are reserved for Section 3.3.2, however, being aware of the purpose and basic structure of a spatial weights matrix is essential in the understanding of the spatial lag and spatial error models, which are introduced next.

### 3.2.2 Spatial Lag Model

When spatial autocorrelation is observed in the dependent variable, it must be controlled for in order to produce reliable parameter estimates (Anselin & Griffith, 1988). To capture this autocorrelation in a regression model an independent variable is added and specified as a spatial lag of the dependent variable. At its most basic formulation the mixed regressive, spatial autoregressive, or spatial lag, model is as follows

$$\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

*Equation 2*

where  $\mathbf{y}$  is an  $n$  by 1 vector of observations of the dependent variable,  $\mathbf{W}\mathbf{y}$  is the corresponding spatially lagged dependent variable vector for the spatial weights matrix  $\mathbf{W}$ ,  $\mathbf{X}$  is an  $n$  by  $k$  matrix of explanatory variables,  $\rho$  is the spatial autoregressive parameter,  $\boldsymbol{\beta}$  is a  $k$  by 1 vector of explanatory parameter estimates, and  $\boldsymbol{\varepsilon}$  is an  $n$  by 1 vector of disturbances assumed  $\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I})$ .

When reformulating the spatial lag model, it can be seen that  $\mathbf{W}\mathbf{y}$  creates a non-zero correlation with the error term and as Anselin and Bera (1998) point out, not only is the spatial lag of observation  $i$  correlated with the error term at  $i$ , it is correlated with the error terms at all locations. This can be seen with the reformulation using Leontief inverse notation

$$\mathbf{y} = (\mathbf{I} - \rho \mathbf{W})^{-1} \mathbf{X}\boldsymbol{\beta} + (\mathbf{I} - \rho \mathbf{W})^{-1} \boldsymbol{\varepsilon}$$

*Equation 3*

The spatial autocorrelation, the covariance of observations across space, is determined by the processes embedded in the spatial lag model and can be seen to equal  $(\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{\Omega}(\mathbf{I} - \rho\mathbf{W}')^{-1}\mathbf{\varepsilon}$ , where  $\mathbf{\Omega}$  is the variance matrix of the error term  $\mathbf{\varepsilon}$  which is assumed to be  $\mathbf{\Omega} = \sigma^2\mathbf{I}$ , resulting in a variance matrix equal to

$$var(\mathbf{y}) = \sigma^2(\mathbf{I} - \rho\mathbf{W})^{-1}(\mathbf{I} - \rho\mathbf{W}')^{-1}$$

*Equation 4*

The resulting matrix is a full matrix, implying that each location is correlated with every other, but in manner that closer locations have more affect than distant ones as defined by the conceptualization of neighbors in  $\mathbf{W}$ , in effect operationalizing Tobler's (1970) first law of Geography. When the resulting variance structure is not accounted for and parameter estimates are obtained using least squares estimation, they are biased and inefficient (Anselin, 1988). Because of this, spatial lag models are usually regressed with a maximum likelihood estimation (Burt, Barber, & Rigby, 2009).

### 3.2.3 Spatial Error Model

A spatial error model is used when spatial autocorrelation is present in the error terms. Much like the spatial lag model, a spatial error model uses a spatial weights matrix to control of the spatial effects of neighboring observances, but unlike the spatial lag model the spatial weights matrix is applied to the error terms, not the dependent variable. The resulting error variance will be such that while unbiased, OLS estimates will be inefficient, thus other estimation techniques are required, with maximum likelihood

estimation being the most prevalent (Anselin & Bera, 1998). The most common specification of the spatial error model is

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

$$\mathbf{u} = \lambda \mathbf{W}\mathbf{u} + \boldsymbol{\varepsilon}$$

*Equation 5*

where  $\mathbf{y}$  is an  $n$  by 1 vector of observations of the dependent variable,  $\mathbf{X}$  is an  $n$  by  $k$  matrix of explanatory variables,  $\boldsymbol{\beta}$  is a  $k$  by 1 vector of explanatory parameter estimates, and  $\mathbf{u}$  being a composite error term where  $\lambda$  is the spatial autoregressive parameter on  $\mathbf{W}\mathbf{u}$ , the corresponding spatially lagged error vector ( $\lambda$  is used to distinguish the notation from the spatial autoregressive term  $\rho$  in the spatial lag model, but their role is identical), and  $\boldsymbol{\varepsilon}$  is an  $n$  by 1 vector of disturbances assumed  $\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I})$ .

Alternately, solving for  $\mathbf{u}$  in the composite error term and plugging that into Equation 5, the spatial error model can be rewritten as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + (\mathbf{I} - \lambda \mathbf{W})^{-1} \boldsymbol{\varepsilon}$$

*Equation 6*

From which we can see that the error variance for the spatial error model is structured identically to the variance of the dependent variable in the spatial lag model. Formally,

$$\text{var}(\boldsymbol{\varepsilon}) = \sigma^2 (\mathbf{I} - \lambda \mathbf{W})^{-1} (\mathbf{I} - \lambda \mathbf{W}')^{-1}$$

*Equation 7*

### 3.2.4 Notation and Ordering of Flows

Let  $\mathbf{Y}$  be an  $m$  by  $n$  matrix of flows from each of the  $n$  origins to each of the  $m$  destinations where each of the  $n$  columns represents a different origin and each of the  $m$  rows represents a different destination. We can create an  $N(=m \times n)$  by 1 vector of these flows in one of two ways, one representing origin-centric ordering and the other representing destination-centric ordering. Starting with  $\mathbf{Y}$ , whose columns represent the origins and rows represent destinations, we can obtain a vector of origin-centric ordering with  $\mathbf{y} = \text{vec}(\mathbf{Y})$  or obtain a vector of destination-centric ordering with  $\mathbf{y} = \text{vec}(\mathbf{Y}')$ . From here out, the following description of the methods will be with origin-centric ordering. Where the first  $m$  elements in the  $N$  by 1 vector  $\mathbf{y}$  represent flows from origin 1 to all  $m$  destinations, applicably, all exports from Alberta to each of the 48 contiguous U.S. states ordered alphabetically from Alabama to Wyoming, and with the last  $m$  elements representing flows from origin  $n$  to all destinations 1 to  $m$ , applicably, all exports from Saskatchewan to each of the 48 contiguous U.S. states ordered alphabetically from Alabama to Wyoming. For this project,  $N$  is equal to 480, representing all 10 Canadian provinces' exports to each of the lower 48 U.S. states.

Now that the appropriate background information on the standard formulation of the spatial weights matrix and spatial lag and error models have been established, each of the three gravity model estimation techniques used in this study; a standard non-spatial gravity model, a spatial lag gravity model, and a spatial error gravity model are described in the following section.

### 3.3 Procedural Methodology

#### 3.3.1 Non-Spatial Gravity Model

The non-spatial gravity model of this study is equivalent to the standard approach used in the literature to estimate gravity models in international trade, which is to use least squares estimation. This is the method used by McCallum (1995), M. A. Anderson and Smith (1999), and Wall (2000) in their analyses of U.S.-Canadian trade. The formulation of the non-spatial gravity model is

$$\mathbf{y} = \alpha \mathbf{1}_N + \beta_o \mathbf{x}_o + \beta_d \mathbf{x}_d + \gamma \mathbf{d} + \boldsymbol{\varepsilon}$$

*Equation 8*

where  $\mathbf{y}$  is an  $N$  by 1 vector of the natural log of trade flows plus 1,  $\mathbf{1}_N$  is an  $N$  by 1 vector of ones with  $\alpha$  denoting its constant parameter term,  $\mathbf{x}_o$  and  $\mathbf{x}_d$  are  $N$  by 1 vectors of the natural log of GDP for each the origin and destination, respectively, with  $\beta_o$  and  $\beta_d$  being their associated scalar parameters,  $\mathbf{d}$  is an  $N$  by 1 vector of the natural log of distance, in miles, between each OD pair with  $\gamma$  being its associated scalar parameter and the error term is represented by the  $N$  by 1 vector  $\boldsymbol{\varepsilon}$  which is assumed  $\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I}_N)$ .

This estimation has five differing specifications, keeping  $\mathbf{x}_o$  and  $\mathbf{x}_d$  constant across all specifications, the dependent variable and distance measure were changed in accordance to Table 1. Each specification was run for both 2006 and 2008, for a total of 10 least squares regressions.

*Table 1. Specifications of the Non-Spatial Gravity Model*

<u>Specifications of the Non-Spatial Gravity Model</u>	
<u>Dependent Variable</u>	<u>Distance Measure</u>
Total exports	Great Circle
Total exports	Network Weighted
Road exports	Road Network
Rail exports	Rail Network
Air exports	Air Network (Great Circle)

These five specifications, specifically the first, were used for the sake of comparison to the spatial lag and spatial error gravity model to what can be considered “standard” gravity model estimations, that is to say, using least squares estimation thus assuming independence of flows and not controlling for spatial dependence, in the dependent variable or disturbances. As discussed earlier, the first specification in Table 1, total trade while using great circle distance is by far the most widely used gravity model specification in the international trade literature and thus will be a main focus of comparison to the spatial lag and spatial error gravity model estimations.

### 3.3.2 Generation of Gravity Model Spatial Weights Matrices

For this study queen contiguity was used to specify the spatial weights matrix for both the U.S. and Canada for use in both spatial lag and spatial error gravity models. Because Prince Edward Island (PEI) does not share a land border with any other provinces, in addition to standard queen contiguity, it was defined as a neighbor of New Brunswick and Nova Scotia. This was done because although PEI does not share a land border with New Brunswick or Nova Scotia, they have similar economies, they are



connected with road networks, and PEI lies less than 30 kilometers from New Brunswick and Nova Scotia. Queen contiguity was chosen because transportation networks are not limited to connecting states and provinces across an edge. Two states that are connected by a transportation network that crosses only at a vertex are just as connected as if that network crossed an edge, therefore only using rook or bishop contiguity does not accurately represent the connectivity of neighboring observations.

The first step in implementing the spatial lag and spatial error gravity models was to construct the spatial weights matrices that follow the structure developed by LeSage and Pace (2008) for both Canada and the U.S., the origin and destination, respectively, and then the third, origin-to-destination spatial weights matrix. This required the generation of a row-standardized queen contiguity 10 by 10 (corresponding to the 10 Canadian provinces under study) origin matrix  $\mathbf{W}_{can}$  and a row-standardized queen contiguity 48 by 48 (corresponding to the 48 contiguous U.S. states under study) destination matrix  $\mathbf{W}_{usa}$  which was done using the spatial weights generation tool on polygon shapefiles of Canada and the U.S. in GeoDa (Anselin et al., 2006). In order for these spatial weights matrices to be operational in an origin-destination application they needed to be expanded so that their dimensions were of equal size, effectively allowing each observation to have corresponding origin and destination weights. LeSage and Pace (2008) show that the Kronecker product of the spatial weights matrix and an identity matrix can be used to accomplish this. In their application each origin was also a destination, allowing for the use of a single spatial weights matrix and an identity matrix equal in size to that of the spatial weights matrix, of size  $n$  by  $n$ , with  $n$  being the number

of regions under study. However, in the case where all origins are not also destinations, as in the case of this research, a slight modification is needed. Instead of the Kronecker product of a single spatial weights matrix and an identity of the same dimensions, it can be seen that the Kronecker product of the origin matrix and an identity matrix with dimensions corresponding to size of the destination spatial weights matrix generate the correct origin spatial weights matrix and the Kronecker product of an identity matrix with dimensions corresponding to size of the origin spatial weights matrix and the destination matrix generate the correct destination spatial weights matrix. Formally,

$$\mathbf{W}_o = \mathbf{W}_{can} \otimes \mathbf{I}_{usa}$$

$$\mathbf{W}_d = \mathbf{I}_{can} \otimes \mathbf{W}_{usa}$$

The resulting  $\mathbf{W}_o$  is an  $N$  by  $N$  row standardized spatial weights matrix used to capture origin-based spatial dependence, operationally, the average of flows from neighbors of the origin to the destination. Similarly,  $\mathbf{W}_d$  is an  $N$  by  $N$  row standardized spatial weights matrix used to capture destination-based spatial dependence, operationally, the average of flows from the origin to neighbors of the destination.

The third type of spatial dependence that was modeled arises from the product of  $\mathbf{W}_o$  and  $\mathbf{W}_d$  and is defined as

$$\mathbf{W}_w = \mathbf{W}_o \mathbf{W}_d$$

The resulting  $\mathbf{W}_w$  is an  $N$  by  $N$  row standardized spatial weights matrix used to capture what LeSage and Pace (2008) call origin-to-destination spatial dependence and reflects the average of flows from neighbors of the origin to neighbors of the destination.

### 3.3.3 Spatial Lag Gravity Model

The spatial lag gravity model that was used in this research is as shown below

$$\mathbf{y} = \rho_o \mathbf{W}_o \mathbf{y} + \rho_d \mathbf{W}_d \mathbf{y} + \rho_w \mathbf{W}_w \mathbf{y} + \mathbf{X}\beta + \boldsymbol{\varepsilon}$$

$$\mathbf{X}\beta = \alpha \mathbf{1}_N + \beta_o \mathbf{x}_o + \beta_d \mathbf{x}_d + \gamma \mathbf{d}$$

$$\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I}_N)$$

*Equation 9*

This model includes the exact same explanatory variables as the non-spatial gravity model, as represented by  $\mathbf{X}\beta$ , with the important addition of the three spatially lagged dependent variables on the right hand side of the equation. In conjunction with their associated scalar parameters;  $\rho_o, \rho_d, \rho_w$ , the three spatial weights matrices indicate the strength of the respective dependence that they represent.

The spatial lag gravity model was estimated with five differing specifications, keeping  $\mathbf{x}_o$ ,  $\mathbf{x}_d$ , and the structure of all three spatial weights matrices constant across all specifications, the dependent variable and distance measure change in accordance to Table 2. Each specification was run for both years 2006 and 2008, for a total of 10 spatial lag gravity model estimations.

*Table 2. Specifications of the Spatial Lag Gravity Model*

Specifications of the Spatial Lag Gravity Model	
<i>Dependent Variable</i>	<i>Distance Measure</i>
Total exports	Great Circle
Total exports	Network Weighted
Road exports	Road Network
Rail exports	Rail Network
Air exports	Air Network (Great Circle)

As previously mentioned least squares estimation is no longer valid in the presence of spatial dependence and maximum likelihood is the most common form of estimation, following this maximum likelihood was used to estimate both the spatial lag and spatial error models.

As the name implies, the objective of maximum likelihood estimation is, given the data, find the set of parameters that maximize the likelihood of the specified likelihood function, or in other words, find the set of parameters that give the highest possible joint probability for the joint density function of the given data and estimated parameters. In practice this likelihood function is logged so to be in the computationally easier additive form. The log-likelihood function for the spatial lag gravity model is as follows

$$\ln L = \ln |\mathbf{I}_N - \rho_o \mathbf{W}_o - \rho_d \mathbf{W}_d - \rho_w \mathbf{W}_w| - \frac{N}{2} \ln(2\pi) - \frac{N}{2} \ln(\sigma^2) - \frac{\boldsymbol{\varepsilon}' \boldsymbol{\varepsilon}}{2\sigma^2}$$

$$\boldsymbol{\varepsilon} = \mathbf{y} - \rho_o \mathbf{W}_o \mathbf{y} - \rho_d \mathbf{W}_d \mathbf{y} - \rho_w \mathbf{W}_w \mathbf{y} - \mathbf{X}\boldsymbol{\beta}$$

*Equation 10*

The first term in the log-likelihood function is the natural log of the determinant of the Jacobian matrix of the transformation. The calculation of this term is the main challenge of estimating the log-likelihood of spatial models, however, because of the sparse nature of spatial weights matrices, sparse matrix routines can be used to more efficiently find the equivalent value (Pace & LeSage, 2010). The method of computing the log determinant of the Jacobian that was used for this research was originally put forth by Pace and Barry (1997) and is calculated using LU decomposition as the sum of the logs of the pivots of U. The second term is a constant that in practice is normally removed. The third and fourth terms reduce and thus the concentrated log-likelihood function for the spatial lag gravity model is as follows

$$\ln L = \ln |\mathbf{I}_N - \rho_o \mathbf{W}_o - \rho_d \mathbf{W}_d - \rho_w \mathbf{W}_w| - \frac{N}{2} \ln(\boldsymbol{\varepsilon}' \boldsymbol{\varepsilon})$$

*Equation 11*

Expanding on  $\boldsymbol{\varepsilon}$  as defined above, it is the  $N$  by 1 column vector error term resulting from a least squares regression on a spatially filtered dependent variable. Thus  $\boldsymbol{\varepsilon}' \boldsymbol{\varepsilon}$  is simply the sum of the squared errors. Spatially filtering the dependent variable can be interpreted as a way to clean the dependent variable of the effects of spatial autocorrelation (Anselin, 2002). The spatial lag gravity model expressed with a spatially filtered dependent variable is as shown below

$$(\mathbf{I}_N - \rho_o \mathbf{W}_o - \rho_d \mathbf{W}_d - \rho_w \mathbf{W}_w) \mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

*Equation 12*

From equation 12 it can be seen that once the dependent variable has been spatially filtered the remaining structure is identical to the non-spatial gravity model.

The spatial lag maximum likelihood estimates for  $\beta$  and  $\sigma^2$  are obtained from the usual first-order conditions, details of which can be found in Anselin (1988). Formally;

$$\beta_{ML} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'(\mathbf{I}_N - \rho_o\mathbf{W}_o - \rho_d\mathbf{W}_d - \rho_w\mathbf{W}_w)\mathbf{y}$$

*Equation 13*

and

$$\sigma_{ML}^2 = \frac{\boldsymbol{\varepsilon}'_{ML}\boldsymbol{\varepsilon}_{ML}}{N}$$

$$\boldsymbol{\varepsilon}_{ML} = (\mathbf{I}_N - \rho_o\mathbf{W}_o - \rho_d\mathbf{W}_d - \rho_w\mathbf{W}_w)\mathbf{y} - \mathbf{X}\beta_{ML}$$

*Equation 14*

### 3.3.4 Spatial Error Gravity Model

The spatial lag gravity model used in this research is as shown below

$$\mathbf{y} = \mathbf{X}\beta + \mathbf{u}$$

$$\mathbf{X}\beta = \alpha\mathbf{1}_N + \beta_o\mathbf{x}_o + \beta_d\mathbf{x}_d + \gamma\mathbf{d}$$

$$\mathbf{u} = \lambda_o\mathbf{W}_o\mathbf{u} + \lambda_d\mathbf{W}_d\mathbf{u} + \lambda_w\mathbf{W}_w\mathbf{u} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} \sim N(0, \sigma^2\mathbf{I}_N)$$

*Equation 15*

This model uses the same explanatory variables as the non-spatial gravity model and is similar to the spatial lag gravity model except that the spatial weights matrices are moved to the error term to create the composite error term  $\mathbf{u}$ . As mentioned early, the

spatial autoregressive terms that in the spatial lag model were notated as  $\rho$ , are notated as  $\lambda$  in the spatial error context to distinguish their formulations, however their role is identical.

The spatial error gravity model was estimated with five differing specifications, keeping  $\mathbf{x}_o$ ,  $\mathbf{x}_d$ , and the structure of all three spatial weights matrices constant across all specifications, the dependent variable and distance were measure changed in accordance to Table 3. Each specification was run for both 2006 and 2008, for a total of 10 spatial error regressions.

*Table 3. Specifications of the Spatial Error Gravity Model*

Specifications of the Spatial Error Gravity Model	
<i>Dependent Variable</i>	<i>Distance Measure</i>
Total exports	Great Circle
Total exports	Network Weighted
Road exports	Road Network
Rail exports	Rail Network
Air exports	Air Network (Great Circle)

As Anselin and Bera (1998) describe, maximum likelihood estimation for the spatial error model can be approached by considering it as a special case of general parameterized nonspherical error terms for which  $var(\mathbf{u}) = \sigma^2 \mathbf{\Omega}(\lambda)$ . Where for the spatial error gravity model

$$\mathbf{\Omega}(\lambda) = [(\mathbf{I}_N - \lambda_o \mathbf{W}_o - \lambda_d \mathbf{W}_d - \lambda_w \mathbf{W}_w)'(\mathbf{I}_N - \lambda_o \mathbf{W}_o - \lambda_d \mathbf{W}_d - \lambda_w \mathbf{W}_w)]^{-1}$$

*Equation 16*

The log-likelihood function for the spatial error gravity model takes the form

$$\ln L = -\frac{1}{2} \ln |\mathbf{\Omega}(\lambda)| - \frac{N}{2} \ln(2\pi) - \frac{N}{2} \ln(\sigma^2) - \frac{(\mathbf{y} - \mathbf{X}\beta)' \mathbf{\Omega}(\lambda)^{-1} (\mathbf{y} - \mathbf{X}\beta)}{2\sigma^2}$$

*Equation 17*

Similar to the log-likelihood function for the spatial lag gravity model, Anselin and Bera (1998) show that Equation 17 can be concentrated to take the form

$$\ln L = \ln |\mathbf{\Omega}(\lambda)| - \frac{N}{2} \ln \left( \frac{\mathbf{\varepsilon}' \mathbf{\varepsilon}}{N} \right)$$

*Equation 18*

where  $\mathbf{\varepsilon}$  is the column vector of error terms from a least squares estimation of the spatially filtered spatial error model. The formulation for the spatially filtered spatial error model differs from the spatially filtered spatial lag model in that the filter is applied on both sides of the equation, to the dependent and explanatory variables. Functionally,

$$(\mathbf{I}_N - \lambda_o \mathbf{W}_o - \lambda_d \mathbf{W}_d - \lambda_w \mathbf{W}_w) \mathbf{y} = (\mathbf{I}_N - \lambda_o \mathbf{W}_o - \lambda_d \mathbf{W}_d - \lambda_w \mathbf{W}_w) \mathbf{X} \beta + \mathbf{\varepsilon}$$

*Equation 19*

The spatial error maximum likelihood estimates for  $\beta$  and  $\sigma^2$  are as follows

$$\beta_{ML} = (\mathbf{X}' \mathbf{\Omega}(\lambda)^{-1} \mathbf{X})^{-1} \mathbf{X}' \mathbf{\Omega}(\lambda)^{-1} \mathbf{y}$$

*Equation 20*

and

$$\sigma_{ML}^2 = \frac{(\mathbf{y} - \mathbf{X} \beta_{ML})' \mathbf{\Omega}(\lambda)^{-1} (\mathbf{y} - \mathbf{X} \beta_{ML})}{N}$$

*Equation 21*



Estimating a gravity model of U.S.-Canadian trade using the three differing methods described in this chapter provide means in which to answer the research goals of this study. Using the non-spatial gravity model acts as a baseline, a method equivalent to previous literature, for comparison to when spatial dependence is accounted for in the estimation procedure by using the spatial lag and spatial error gravity models, the results of which are discussed in the following chapter.

## **Chapter 4**

### **Results**

The resulting parameter estimates varied across estimation technique and across mode of transportation; however, the results for the year 2008 were qualitatively equivalent to 2006. In light of this the discussion below will be centered only on the results for the year 2006. Although the exact numbers mentioned below are for the year 2006, all inferences, trends, and generalizations apply to 2008 as well. All equivalent tables for 2008 are included in the Appendix.

#### **4.1 Distance Measurement Results**

When compared to network distance, great circle distance proved to underestimate the separation between any given province and state pair by an average of 20%. Seen in Table 4, this was true for both the road and rail networks, as well as, the network weighted distance measure.

*Table 4. Network Distance Measurement Increase*

Average Percent Distance Increase to a U.S. State When Using Network Distance			
	Road	Rail	Network Weighted Total
Alberta	24.5	27.1	23.2
British Columbia	23.0	25.6	23.1
Manitoba	22.7	21.6	21.7
New Brunswick	19.0	19.4	18.7
Newfoundland and Labrador	20.9	-	16.8
Nova Scotia	24.8	33.2	25.0
Ontario	23.8	22.6	21.6
Prince Edward Island	19.6	-	13.4
Quebec	16.8	15.3	14.3
Saskatchewan	21.1	26.5	22.1
Overall Average	20.7	21.0	20.0

Note: NL & PEI do not have rail estimates because there are no connecting rail networks

## 4.2 Non-Spatial Gravity Model Results

The results for all least squares estimations are below in Table 5. The standard approach used by previous literature, total trade and great circle distance, estimates the influence of the origin GDP and destination GDP to be roughly equivalent and yielded a distance effect of -1.58, an estimate that suggests given a 10% increase in distance exports from Canadian provinces to U.S. states should decrease by approximately 15.8%. When using the network weighted distance, the distance effect increased, that is became more negative, to -1.65 for total exports while origin and destination GDP parameter estimates essentially remained unchanged. When total trade was disaggregated into the different modes of transportation, larger differences emerged.

Exports transported across roads saw a jump in the distance effect to -1.76 with a lessening influence of origin and destination GDP. Exports shipped via rail saw a

distance effect around -0.91 with the origin GDP playing a more influential role than that of the destination GDP. Air exports saw the smallest distance effect estimate, a trend that continues across all estimation techniques, at -0.35 with the origin GDP parameter estimate being larger than that of the destination.

*Table 5. Non-Spatial Gravity Model Results*

Non-Spatial Gravity Model Results - 2006					
	Total- great circle distance	Total	Road	Rail	Air
Intercept	<b>-7.3204</b>	<b>-6.5002</b>	<b>-4.3473</b>	<b>-11.2553</b>	<b>-12.1909</b>
<i>Standard Error</i>	0.9100	0.9318	0.7369	1.0740	0.8407
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Origin GDP	<b>1.0144</b>	<b>1.0206</b>	<b>0.9551</b>	<b>1.0503</b>	<b>0.7946</b>
<i>Standard Error</i>	0.0354	0.0351	0.0279	0.0402	0.0327
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Destination GDP	<b>0.9697</b>	<b>0.9660</b>	<b>0.8522</b>	<b>0.7516</b>	<b>0.6193</b>
<i>Standard Error</i>	0.0471	0.0469	0.0372	0.0533	0.0435
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Distance	<b>-1.5747</b>	<b>-1.6521</b>	<b>-1.7607</b>	<b>-0.9118</b>	<b>-0.3543</b>
<i>Standard Error</i>	0.0818	0.0849	0.0662	0.0976	0.0756
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Residual Moran's I	0.3543	0.3467	0.4678	0.4268	0.5677
Sigma-Square	1.1027	1.0924	0.6895	1.4119	0.9411

When using Moran's I to test for the presence of spatial dependence in the observed trade flows, the dependent variables from across all modes of transportation exhibited significant levels of positive spatial autocorrelation. As seen in Table 6, exports transported via road had the highest observed Moran's I value, 0.771 for 2006, while total trade exhibited the lowest, a still very high 0.695. The presence of such large amounts of spatial autocorrelation in the dependent variable suggests the use of least squares

estimation will produce biased and inefficient parameter estimates (Anselin, 1988), thus providing the empirical justification to implement the spatial lag model.

*Table 6. Moran's Index – Dependent Variable*

Moran's Index of Spatial Autocorrelation - Dependent Variable		
	2006	2008
Total exports	0.695	0.682
Road exports	0.771	0.768
Rail exports	0.729	0.714
Air exports	0.723	0.707

### **4.3 Spatial Lag Gravity Model Results**

Results of the spatial lag gravity model can be seen in Table 7. When controlling for spatial effects in the dependent variable by using the spatial lag gravity model, dramatic differences arise in the parameter estimates, of particular interest are the changes to the distance coefficient.

Table 7. Spatial Lag Gravity Model Results

Spatial Lag Gravity Model Results - 2006					
	Total- great circle distance	Total	Road	Rail	Air
Intercept	<b>-5.7879</b>	<b>-5.2426</b>	<b>-2.6445</b>	<b>-6.6478</b>	<b>-8.4372</b>
<i>Standard Error</i>	1.1889	1.2140	0.9044	1.5357	1.0365
<i>p-value</i>	0.0000	0.0000	0.0036	0.0000	0.0000
Logged Origin GDP	<b>0.6514</b>	<b>0.6627</b>	<b>0.5386</b>	<b>0.5068</b>	<b>0.3600</b>
<i>Standard Error</i>	0.0491	0.0484	0.0383	0.0663	0.0521
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Destination GDP	<b>0.7350</b>	<b>0.7319</b>	<b>0.5920</b>	<b>0.4888</b>	<b>0.5190</b>
<i>Standard Error</i>	0.0566	0.0565	0.0433	0.0619	0.0387
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Distance	<b>-1.0454</b>	<b>-1.1055</b>	<b>-1.0991</b>	<b>-0.4727</b>	<b>-0.1276</b>
<i>Standard Error</i>	0.1100	0.1133	0.0817	0.1466	0.0999
<i>p-value</i>	0.0000	0.0000	0.0000	0.0014	0.2022
<b>W<sub>o</sub></b>	<b>0.2282</b>	<b>0.2299</b>	<b>0.2939</b>	<b>0.2647</b>	<b>0.0962</b>
<i>Standard Error</i>	0.0381	0.0380	0.0364	0.0392	0.0395
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0151
<b>W<sub>d</sub></b>	<b>0.3514</b>	<b>0.3431</b>	<b>0.4457</b>	<b>0.5153</b>	<b>0.5712</b>
<i>Standard Error</i>	0.0443	0.0446	0.0383	0.0391	0.0346
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
<b>W<sub>w</sub></b>	<b>-0.2248</b>	<b>-0.2210</b>	<b>-0.3959</b>	<b>-0.2820</b>	<b>-0.2195</b>
<i>Standard Error</i>	0.0442	0.0442	0.0379	0.0456	0.0448
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Residual Moran's I	0.0942	0.0948	0.1007	-0.0073	0.0109
Sigma-Square	0.9008	0.8968	0.4571	0.9423	0.5476

When using the spatial lag gravity model, across all specifications the influence of origin and destination GDPs saw a reduced influence when compared to their least squares counterparts. The same holds for the distance effect estimates, as variations that were previously attributed to GDPs and distance in the OLS estimation are now accounted for in the parameter estimates of the spatial weights matrices. The distance coefficient for total trade using the network weighted distance dropped from -1.65 for least squares to -1.11 in the spatial lag estimation, a 33% decrease in the estimate of the distance effect. The distance effect for road exports dropped by 38% to -1.10 when compared to the least squares equivalent. For exports transported by rail, the distance effect fell to -0.47 a drop of 48% compared to least squares estimation. Interestingly, when accounting for spatial dependence in air exports, not only does the distance effect become less influential, it fails to remain statistically significant at alpha levels less than .2.

The inclusion of all three spatial weights matrices proved statistically significant in all specifications of the spatial lag gravity model.  $\mathbf{W}_o$ , used to capture origin based spatial dependence, or the average of flows from neighbors of the origin to a particular destination, had a consistent parameter estimate for total, road, and rail exports laying between 0.23 and 0.29. However, for air exports  $\mathbf{W}_o$  fell to a low, yet still significant, 0.10. Destination based spatial dependence, or the average of flows from a particular origin to neighbors of a destination, represented by  $\mathbf{W}_d$ , displayed the largest absolute influence of any of the three spatial weights matrices across all specifications. For total exports the parameter estimate for  $\mathbf{W}_d$  was 0.34, 33% larger than that of  $\mathbf{W}_o$ . Exports

transported by road saw  $\mathbf{W}_d$  become more influential than in total exports, with its parameter estimate being 0.45. While rail and air exports saw an even larger influence of destination based spatial dependence with parameter estimates for  $\mathbf{W}_d$  being 0.52 and 0.57, respectively. Origin-to-destination based spatial dependence, captured by  $\mathbf{W}_w$  and representing the average of flows from the neighbors of the origin to neighbors of the destination were all negatively signed ranging from -0.22 for total and air exports to -0.28 for rail exports, leaving highway exports with the most influential  $\mathbf{W}_w$  estimate at -0.40.

#### **4.4 Spatial Error Gravity Model Results**

The results of the spatial error gravity model can be seen below in Table 8. As expected when modeling spatial dependence in the error terms, the coefficient estimates for the intercept, both GDPs, and the distance parameter in the spatial error gravity model are asymptotically equivalent to those obtained from the non-spatial gravity model. For total trade, the distance coefficient was -1.67, which increased to -1.78 for exports shipped via road. For rail exports the distance effect estimate was -0.99, suggesting with a 10% increase in distance, trade should decrease by an equivalent percentage. Lastly, the distance effect was a relatively low estimate of -0.21 for air exports, which in fact fails to remain statistically different from zero with an alpha value any less than .09.



Table 8. Spatial Error Gravity Model Results

Spatial Error Gravity Model Results - 2006					
	Total- great circle distance	Total	Road	Rail	Air
Intercept	<b>-7.7452</b>	<b>-7.2426</b>	<b>-5.1870</b>	<b>-10.3159</b>	<b>-13.5127</b>
Standard Error	1.4996	1.5267	1.2489	1.7798	1.2985
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Origin GDP	<b>0.9885</b>	<b>0.9974</b>	<b>0.9529</b>	<b>1.0427</b>	<b>0.7972</b>
Standard Error	0.0641	0.0625	0.0581	0.0725	0.0683
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Destination GDP	<b>1.0639</b>	<b>1.0616</b>	<b>0.9331</b>	<b>0.7247</b>	<b>0.6431</b>
Standard Error	0.0492	0.0494	0.0375	0.0592	0.0352
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Distance	<b>-1.6341</b>	<b>-1.6727</b>	<b>-1.7751</b>	<b>-0.9924</b>	<b>-0.2110</b>
Standard Error	0.1507	0.1529	0.1224	0.1758	0.1276
p-value	0.0000	0.0000	0.0000	0.0000	0.0988
<b>W<sub>o</sub></b>	<b>0.1298</b>	<b>0.1352</b>	<b>0.1877</b>	<b>0.2303</b>	<b>0.0328</b>
Standard Error	0.0436	0.0436	0.0435	0.0423	0.0413
p-value	0.0031	0.0020	0.0000	0.0000	0.4269
<b>W<sub>d</sub></b>	<b>0.4894</b>	<b>0.4747</b>	<b>0.6164</b>	<b>0.5169</b>	<b>0.6557</b>
Standard Error	0.0453	0.0461	0.0373	0.0426	0.0334
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
<b>W<sub>w</sub></b>	<b>0.0216</b>	<b>0.0371</b>	<b>-0.1255</b>	<b>-0.0797</b>	<b>-0.0402</b>
Standard Error	0.0679	0.0686	0.0657	0.0685	0.0585
p-value	0.7500	0.5894	0.0566	0.2453	0.4923
Residual Moran's I	-0.0240	-0.0222	-0.0441	-0.0492	-0.0750
Sigma-Square	0.8075	0.8056	0.4072	0.9156	0.5060

Consistent with spatial lag gravity model, out of the three spatial weights matrices,  $\mathbf{W}_d$  proved to be the most influential in the spatial error gravity model. For total trade, destination based spatial dependence captured by  $\mathbf{W}_d$  was 0.48. However, when trade was disaggregated into mode of transportation the influence of  $\mathbf{W}_d$  became even larger. For road exports the coefficient estimate was 0.62, rail exports saw an estimate of 0.52, and air exports had the largest estimate for  $\mathbf{W}_d$  at 0.66. Origin based spatial dependence,  $\mathbf{W}_o$ , had relatively low estimates but were statistically significant in all cases, except in the case of air exports. Origin-to-destination dependence captured by  $\mathbf{W}_w$  showed no statistical significance in any specification of the spatial error gravity model, with each estimate being well within the 95% confidence interval of the null hypothesis that the parameter coefficient is equal to zero.

## **Chapter 5**

### **Discussion and Conclusion**

Disaggregating trade flows into mode of transportation and using their corresponding network distances in spatial econometric estimation techniques allowed for the investigation of how these measures impact the distance effect in international trade when compared to the standard approach of using least squares on total trade and great circle distance. The results found in this study justified all of the original purposes of investigation, although to varying degrees.

#### **5.1 Distance Effect Estimates across Differing Measures of Distance**

When transportation networks were used to create a network weighted distance for each OD pair in an effort to answer if the distance effect would change when using a more realistic measure of distance, the parameter estimate measuring the distance effect was found to increase by less than 5%, to -1.65, from the standard approach which yielded a distance effect of -1.58, each being within one standard error of the other. Thus, using the network weighted distance had no significant impact on the distance effect estimate.

At first glance this result is somewhat surprising given the fact that on average great circle distance underestimated the distance between any given OD pair by 20% when compared to the network weighted distance measure. However, this outcome made apparent that when estimating the effect distance has on trade, what matters is not the individual distance that separates a particular origin from a destination, but how that distance fits into the spatial structure of all origins and destinations. This is to say, what

matters is not so much absolute distance, but relative distance. While great circle distance proved to underestimate the absolute distance that separated any particular origin and destination, it did so in a manner as to underestimate the absolute distance separating that origin from all other destinations by an equal percentage. Therefore when the more accurate measure of network weighted distance was used, the distance between each origin and destination increased absolutely, yet remained unchanged relatively.

This confirms the observation of Disdier and Head (2008) who suggested that distance effect estimates from a small number of papers that analyzed sea transportation routes showed no significant differences from the estimates obtained in studies that use great circle distance. To this author's knowledge, this is the first study to empirically validate this observation.

## **5.2 Distance Effect Estimates across Modes of Transportation**

When total trade was disaggregated into differing modes of transportation, their resulting distance effect estimates varied significantly. Across all estimation procedures; the non-spatial gravity model, the spatial lag gravity model, and the spatial error gravity model, exports transported by road saw the highest distance effect estimate, followed by rail, and then followed by a much less influential distance effect estimate for air exports.

To a certain degree this pattern may in part be due from constraints of the transportation networks themselves. Varying degrees of network accessibility generate heterogeneous distance increases across provinces, potentially making transport across one, relatively sparse network more difficult than across another denser network for any given province. On the whole, however, the distance effect estimates observed for each

transportation network might best be explained as a reflection of the goods that are transported across them.

The mode of transportation used to ship a certain good can be thought of as being chosen based on a function of time and cost effectiveness, with cost effectiveness itself being a measure of many inputs such as; the distance to the destination, the total volume of a single shipment, the value to weight ratio of the unit good, security of the shipment, among many other considerations. This function is able to explain transport mode choice insofar as the mode which is able to transport a good in the most cost effective manner so that it still arrives at the destination within an acceptable timeframe will almost always be the mode of transportation chosen for the shipment of that good. As a result, the shipments that do move across a certain mode of transportation tend to do so for similar reasons. Taking road transport as a baseline, in general goods transported across rail tend to have a lower value to weight ratio and individual shipments are accordingly of larger volume to enable cost effective shipment, while goods transported by air tend to have a much higher value to weight ratio and promptness of delivery is of the upmost importance. The data revealed that when aberrantly large amounts of rail or air exports were observed to move from a particular province to a particular state this tended to be indicative of intra industry trade, which more often than not was dominated by one specific industry common to that provincial-state pair. Supporting the Linder hypothesis (1961), that exports tend to reflect the home market thus trade arises between economic entities with similar market structures, when coupled with the spatial origination and distribution of U.S. industries, this resulted in economic closeness suppressing the

influence of physical distance, thus causing distance effect estimates for rail and air exports that were less influential than those for road or total exports and in the case of air exports, dramatically so.

For example, following gravity model logic, Québec should trade more with Michigan than Tennessee, given that Michigan is much closer to Québec and has a larger economic mass, measured by Gross State Product, than that of Tennessee; and for exports transported by road this logic holds. However, home to the Canadian aluminum industry, in 2008, of the entirety of Québec's U.S. bound exports transported by rail, 42.2% were categorized under HTS code 76, "Aluminum and Articles", of which 46% of those were exported to a single state, Tennessee, one of the largest producers of aluminum products in the United States. Making Tennessee the destination for a full 19.2% of all U.S. bound rail exports from Québec, more than double the second leading destination of Michigan. Along with this example other observations, such as 56.4% of rail exports from Saskatchewan being HTS code 31, "Fertilizers", of which roughly 60% are exported to only three U.S. states; Illinois, Iowa, and Minnesota, concentrate such a large percentage of all rail shipments to states that are situated far enough from Canada, as to lower the estimate that distance has on their trade when compared to road transport.

The distance effect estimate for Canadian exports transported by air proved to be the smallest of all modes of transportation. As mentioned, for a good to be transported by air it generally needs to have a high value to weight ratio and the expedience of its delivery is very important. For Canadian exports to the U.S., this corresponded to intra industry trade within the high-tech manufacturing industry. Anchored by California and

Texas, trade within the high-tech manufacturing industry consisted of over 60% of all Canadian air exports in 2008. With the presence of such a large percentage of the U.S. high-tech manufacturing industry, California and Texas alone accounted for over 30% of all Canadian air exports, resulting in a distance effect estimate dramatically less influential than that of road or rail.

### **5.3 Distance Effect Estimates and Spatial Dependence**

When using the spatial lag gravity model to control for spatial dependence observed in trade flows, the distance effect estimate decreased significantly, on average by over a third, as influence previously attributed to the distance parameter is now attributed to the spatial autoregressive parameters. Destination-based spatial dependence proved to have the greatest influence of all three of the spatial weights matrices. Intuitively, this makes sense given the average origin province is much larger than the average destination state, allowing for a more diverse set of trade flows originating from a neighboring origin to a certain destination than trade flows arriving at a neighboring destinations from one certain origin.

All  $\beta$  estimates for the spatial error gravity model were asymptotically equivalent to those generated by least squares estimation, as is expected for a properly specified spatial error model. The empirical gains are realized in that the spatial error model reduced the error variance thus strengthening the inferential power of the overall model.

Empirical rationale for using the spatial econometric gravity models can easily be seen from the levels of spatial autocorrelation in the observed trade flows and their resulting least squares error terms, however theoretical rationale would provide even

stronger reasoning for their implementation. Curry (1972) is credited with being the first to conceptualize the problem of spatial dependence in flow observations who hypothesized that distance effects were being confounded by the presence of spatial autocorrelation. Griffith and Jones (1980) note in their journey to work study that flows from an origin are enhanced or diminished by influence of neighboring origins and reiterate the same for destinations and their neighbors. These studies focus on OD flows of migration data, however, and while supporting spatial econometric gravity modeling in general, they do not speak to theoretical motivations in respect to international trade. In this respect there has been little theoretical work that explicitly includes spatial dependence in deriving the occurrence of international trade. One exception is the work of Behrens, Ertur, and Koch (2012) who extend the work of J. E. Anderson and Wincoop (2004) . They use monopolistic competition coupled with a CES utility function to derive a gravity model for trade flows that contain spatial lags of the dependent variable.

As to which spatial gravity model is theoretically suited to model this interdependence of trade flows, depends on how one interprets the proper use of the spatial lag and spatial error models.

It is not disputed that a spatial lag model is the proper specification when the values of the dependent variable at one location cause a change in the value of the dependent variable at a neighboring location. The discrepancy arises when one infers the definition of cause. In a strict sense, it is not intuitive to assume that because Ontario sends a large amount of exports to Michigan that these exports themselves cause more exports to be sent to Ohio. It makes much more sense that similar industries, natural



resource needs, and similar distance might be the actual underlying cause for similar levels of exports from Ontario to these two states. When interpreted in this manner, the spatial error model becomes the theoretically proper model for estimation. However, the majority of spatial econometric literature, including the relevant works of Behrens et al. (2012), LeSage and Pace (2008), and LeSage and Fischer (2010), interpret cause in a more lenient fashion and conclude that the spatial lag specification is theoretically appropriate. Thus whether the spatial lag gravity model or spatial error gravity model is the most appropriate is dependent upon one's definition of causality.

## 5.4 Conclusion

This study investigated how distance effect estimates from standard gravity model analysis respond when a more accurate measure of distance is employed and when they are estimated using techniques that take into account spatial autocorrelation. It was found that while the standard distance measure of great circle distance on average underestimated the distance between origins and destinations by 20%, it did so consistently enough across all origin-destination pairs, as to keep the relative separation of origins and destinations the same resulting in a distance effect estimate that remained unchanged.

When trade flows were disaggregated into mode of transportation, differences in the distance effect estimate emerged. Canadian exports transported across road saw the highest distance effect estimate, followed by rail, and with exports transported by air seeing little to no influence from the distance effect. Differences in distance effect estimates across modes of transportation are thought to be a reflection of each mode's most efficient use. With air transportation being used to transport high value to weight ratio goods across long distances, rail transport used for medium to long range delivery for lower value to weight ratio goods which are moved in large volumes at one time, and leaving road transport for goods that are shipped relatively short distances and generally goods not efficiently moved across other modes of transport. Inflated by the fact that the majority of automotive industry trade from Ontario to Michigan is transported across highways, this leaves road transported exports with the highest distance effect estimates for any mode of transportation.

When spatial dependence was controlled for with the use of spatial lag gravity model and spatial error gravity model both models displayed improved overall performance as measured by sigma-square. The spatial lag gravity model estimated the distance effect to be much lower than least squares estimates, as influence previously attributed to the distance parameter were spread to the spatial autoregressive terms, while spatial error gravity model estimates of the distance effect were expectedly asymptotically equal. The assumption of independence of flows as long been questioned, providing motivation for an estimation technique that takes into account the spatial dependence inherent in these flows, but as to which model is most theoretically appropriate, depends on how one interprets the appropriate use of the spatial lag and spatial error models.

Further research in spatial econometric gravity model analysis can be taken in many directions. Network distance provided no advantage over great circle distance in estimating the distance effect for this study, however, this can hardly be assumed to hold in all cases. In areas of the world where transportation networks are not as dense and highly connected as throughout all of the U.S. and Canada, heterogeneous network accessibility can create a situation where the use of transportation network distance does provide an advantage over great circle distance measurements. Future research should investigate the sensitivity of distance effect estimates to the level of systematic bias in great circle distance measurements. Measures such as the spatial autocorrelation of distance measurement errors may be able to better inform when it would be advantageous to use transportation network distance over great circle distance.

Since to a large degree the mode of transport that a good moves across seems to be most influenced by the good itself, further research should look into distance effect estimates when trade is disaggregated by HTS or NAICS codes. Disaggregating trade by industry could provide interesting insight into the spatial patterns of U.S. and Canadian economic activity and allow one to see if distance has more of an influence in one industry over another.

Finally, the role of spatial dependence in international trade flows is relatively new to explicit investigation and accordingly there many areas which remain largely unexplored. Future research should focus on extending spatial econometric modeling in empirical international trade analysis. However, more importantly, more work needs to be done in developing stronger theoretical justifications for including spatial dependence in the modeling international trade.

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## Appendix

Table 9. Non-Spatial Gravity Model Results – 2008

Non-Spatial Gravity Model Results - 2008					
	Total- great circle distance	Total	Road	Rail	Air
Intercept	<b>-7.1890</b>	<b>-6.3445</b>	<b>-4.0570</b>	<b>-10.5883</b>	<b>-10.4058</b>
<i>Standard Error</i>	0.9641	0.9852	0.7857	1.1175	0.9374
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Origin GDP	<b>0.9961</b>	<b>1.0031</b>	<b>0.9432</b>	<b>1.0386</b>	<b>0.7113</b>
<i>Standard Error</i>	0.0374	0.0371	0.0297	0.0417	0.0363
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Destination GDP	<b>0.9834</b>	<b>0.9797</b>	<b>0.8281</b>	<b>0.7632</b>	<b>0.6126</b>
<i>Standard Error</i>	0.0504	0.0501	0.0401	0.0561	0.0490
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Distance	<b>-1.6110</b>	<b>-1.6923</b>	<b>-1.7805</b>	<b>-1.0208</b>	<b>-0.4956</b>
<i>Standard Error</i>	0.0862	0.0894	0.0702	0.1011	0.0838
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Residual Moran's I	0.3676	0.3576	0.5268	0.4043	0.5976
Sigma-Square	1.2263	1.2129	0.7777	1.5192	1.1593

Table 10. Spatial Lag Gravity Model Results – 2008

Spatial Lag Gravity Model Results - 2008					
	Total- great circle distance	Total	Road	Rail	Air
Intercept	<b>-5.5495</b>	<b>-5.0197</b>	<b>-2.4284</b>	<b>-5.9906</b>	<b>-7.1785</b>
<i>Standard Error</i>	1.2831	1.3081	0.9609	1.6333	1.1936
<i>p-value</i>	0.0000	0.0001	0.0118	0.0003	0.0000
Logged Origin GDP	<b>0.6059</b>	<b>0.6175</b>	<b>0.4913</b>	<b>0.4875</b>	<b>0.2885</b>
<i>Standard Error</i>	0.0536	0.0528	0.0416	0.0697	0.0610
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Destination GDP	<b>0.7233</b>	<b>0.7208</b>	<b>0.5630</b>	<b>0.4675</b>	<b>0.5028</b>
<i>Standard Error</i>	0.0607	0.0606	0.0455	0.0675	0.0424
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Distance	<b>-1.0223</b>	<b>-1.0822</b>	<b>-1.0478</b>	<b>-0.5172</b>	<b>-0.1946</b>
<i>Standard Error</i>	0.1186	0.1219	0.0871	0.1546	0.1155
<i>p-value</i>	0.0000	0.0000	0.0000	0.0009	0.0926
<b>W<sub>o</sub></b>	<b>0.2476</b>	<b>0.2485</b>	<b>0.3053</b>	<b>0.3103</b>	<b>0.1087</b>
<i>Standard Error</i>	0.0381	0.0380	0.0362	0.0379	0.0384
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0049
<b>W<sub>d</sub></b>	<b>0.3874</b>	<b>0.3790</b>	<b>0.4955</b>	<b>0.5270</b>	<b>0.6093</b>
<i>Standard Error</i>	0.0431	0.0435	0.0365	0.0389	0.0327
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
<b>W<sub>w</sub></b>	<b>-0.2501</b>	<b>-0.2457</b>	<b>-0.4231</b>	<b>-0.3146</b>	<b>-0.2182</b>
<i>Standard Error</i>	0.0445	0.0445	0.0384	0.0450	0.0455
<i>p-value</i>	0.0000	0.0000	0.0000	0.0000	0.0000
Residual Moran's I	0.0714	0.0705	0.1076	-0.0091	-0.0191
Sigma-Square	0.9598	0.9560	0.4689	0.9709	0.6268

Table 11. Spatial Error Gravity Model Results – 2008

Spatial Error Gravity Model Results - 2008					
	Total- great circle distance	Total	Road	Rail	Air
Intercept	<b>-7.6490</b>	<b>-7.1953</b>	<b>-4.8668</b>	<b>-9.3613</b>	<b>-11.7758</b>
Standard Error	1.5625	1.5857	1.3985	1.8991	1.5168
p-value	0.0000	0.0000	0.0005	0.0000	0.0000
Logged Origin GDP	<b>0.9796</b>	<b>0.9901</b>	<b>0.9584</b>	<b>1.0465</b>	<b>0.7317</b>
Standard Error	0.0687	0.0669	0.0680	0.0731	0.0793
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Destination GDP	<b>1.0655</b>	<b>1.0618</b>	<b>0.9262</b>	<b>0.7159</b>	<b>0.6342</b>
Standard Error	0.0535	0.0537	0.0373	0.0659	0.0392
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Logged Distance	<b>-1.6606</b>	<b>-1.6923</b>	<b>-1.8570</b>	<b>-1.1348</b>	<b>-0.3703</b>
Standard Error	0.1548	0.1568	0.1355	0.1852	0.1483
p-value	0.0000	0.0000	0.0000	0.0000	0.0129
<b>W<sub>o</sub></b>	<b>0.1639</b>	<b>0.1666</b>	<b>0.1890</b>	<b>0.2953</b>	<b>0.0652</b>
Standard Error	0.0433	0.0433	0.0440	0.0403	0.0399
p-value	0.0002	0.0001	0.0000	0.0000	0.1034
<b>W<sub>d</sub></b>	<b>0.5081</b>	<b>0.4926</b>	<b>0.6685</b>	<b>0.4923</b>	<b>0.6705</b>
Standard Error	0.0439	0.0446	0.0341	0.0454	0.0319
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
<b>W<sub>w</sub></b>	<b>-0.0521</b>	<b>-0.0396</b>	<b>-0.1216</b>	<b>-0.0810</b>	<b>-0.0424</b>
Standard Error	0.0676	0.0684	0.0641	0.0695	0.0574
p-value	0.4407	0.5633	0.0587	0.2445	0.4606
Residual Moran's I	-0.0380	-0.0359	-0.0491	-0.0356	-0.0900
Sigma-Square	0.8757	0.8783	0.4018	0.9460	0.5854



### **Vita**

Jesse Piburn was born in Gallatin, TN to Bill and Mary Jane Piburn on November 23, 1988. After graduating from Gallatin High School in May 2007, Jesse attended the University of Tennessee-Knoxville receiving his B.A. in Geography in December 2010. Jesse entered the M.S. degree program in Geography at the University of Tennessee-Knoxville in August 2011 and is scheduled to fulfill the requirements for degree conferment in May 2013.